Shading

Color Interpolation

- **Flat shading**

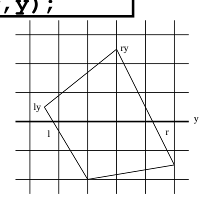
```
glBegin(GL_TRIANGLE);
  glColor3f(...);
  glVertex3f(...);
  glVertex3f(...);
  glVertex3f(...);
glEnd();
```

- **Gouraud shading**

```
glBegin(GL_TRIANGLE);
  glColor3f(...); glVertex3f(...);
  glColor3f(...); glVertex3f(...);
  glColor3f(...); glVertex3f(...);
glEnd();
```


Crow's Algorithm for Convex Polygons

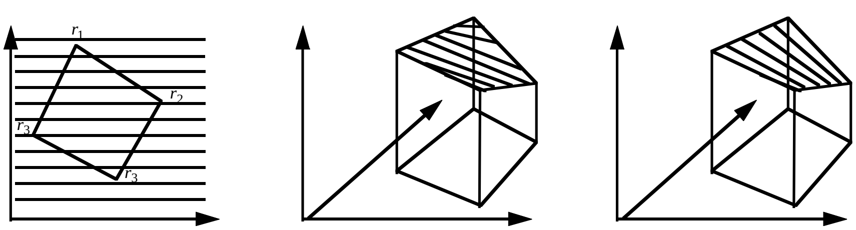
```
scany( vert v[], int n, int bot)
  li = ri = bot;
  ly = ry = y = ceil(v[bot].y);
  for( rem = n; rem > 0 ; ) {
    while( ly <= y && rem-- > 0 ) {
      i = li; li = mod(li-1,n); ly = ceil(v[li].y);
      if( ly > y )
        differencey(&v[i],&v[li],&l,&dl,y);
    }
    while( ry <= y && rem-- > 0 ) {
      i = ri; ri = mod(ri+1,n); ry = ceil(v[ri].y);
      if( ry > y )
        differencey(&v[i],&v[ri],&r,&dr,y);
    }
    for( ; y < ly && y < ry; y++ ) {
      scanx(&l,&r,y);
      increment(&l,&dl);
      increment(&r,&dr);
    }
```



Crow's Algorithm for Convex Polygons

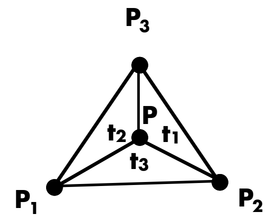
```
scanx( vert *l, vert *r, int y)
  lx = ceil(l->x);
  rx = ceil(r->x);
  if( lx < rx ) {
    differencex( l, r, &s, &ds, lx );
    for( x=lx; x<rx; x++ ) {
      point(x,y);
      increment(&s,&ds);
    }
  }

differencex(vert *l,vert *r, vert *s,vert *ds, int x)
{
  ds->v = (r->v - l->v)/ (r->x - l->x);
   s->v = l->r + (x - l->x) * de->v;
}
differencey(vert *b,vert *t, vert *e,vert *de, int y)
{
  de->v = (t->v - b->v)/ (t->y - b->x);
   e->v = b->v + (x - b->x) * de->v;
}
```


Gouraud Shading



Triangle Interpolation

$$x = x_1t_1 + x_2t_2 + x_3t_3$$
$$y = y_1t_1 + y_2t_2 + y_3t_3$$
$$w = w_1t_1 + w_2t_2 + w_3t_3$$
$$r = r_1t_1 + r_2t_2 + r_3t_3$$
$$g = g_1t_1 + g_2t_2 + g_3t_3$$
$$b = b_1t_1 + b_2t_2 + b_3t_3$$
$$a = a_1t_1 + a_2t_2 + a_3t_3$$
$$\ldots$$



Polygon Interpolation

Interpolate other quantities

- **Phong shading**
  - Interpolate normal
  - Compute new color at each pixel
- **Z-buffering**
  - Interpolate z
  - Resolve hidden surfaces at each pixel
- **Texture mapping**
  - Interpolate texture coordinates
  - Lookup color at that position in a texture map/image

Texture Maps

Map an image to a surface

Q. How is texture mapped to the surface?

- **Dimensionality**
- **2D Texture coordinates (s,t)**
  - **Surface parameters (u,v)**
  - **Direction: reflection vectors**
  - **Projection: decal map**

Q. What does texture control?

- **Surface color and opacity**
- **Reflection functions**
- **Illumination functions**
- **Perturb geometry**

Texture Mapping

2D Texture Space

↓ **Parameterization**

3D Object Space

↓ **Model Transformation**

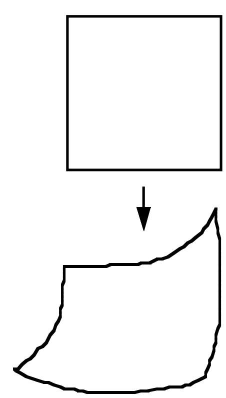
3D World Space

↓ **Viewing Transformation**

3D Camera Space

↓ **Projection**

2D Image Space



Mapping Algorithms

Forward mapping (video effects)

```
for all (u,v)
  (x,y) = f(u,v)
  image[x][y] = texture[u][v]
```

Backward mapping (workstations)

```
for all (x,y)
  (u,v) = f-1(x,y)
  image[x][y] = texture[u][v]
```


Polygonal Mapping

Parameterization may be written as a linear map

$$P = P_0 + uP_1 + vP_2$$

The transformation from texture space to image space be performed with a single 2d projective matrix

$$(x, y, w) = P((V(M(T(u, v, 1))))) = (PVMT)(u, v, 1) \qquad \begin{pmatrix} x \\ y \\ w \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

The inverse transformation from image space to texture space is also a 2d projective matrix

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

Incremental Texture Mapping Loop

```
x = xmin;
unum = A*x+B*y+C;
vnum = D*x+E*y+F;
den  = G*x+H*y+I;
for( ; x<xmax; x++ ) {
  image[x][y]=texture(unum/den,vnum/den);
  unum += A;
  vnum += D;
  den  += G;
}
```

$$u = \frac{unum}{den} = \frac{Ax+By+C}{Gx+Hy+I}$$

$$v = \frac{vnum}{den} = \frac{Dx+Ey+F}{Gx+Hy+I}$$

$$x' = \frac{x}{w} = \frac{au+bv+c}{gu+hv+i}$$

$$y' = \frac{y}{w} = \frac{du+ev+f}{gu+hv+i}$$

Linearly Interpolating Texture Coords

Generalized Interpolation

$$(x,y,z,1,r,g,b,a,u,v,...,1)$$

Parameters affinely related $\downarrow$ **Transform and Clip**

$$(xw,yw,zw,w,r,g,b,a,u,v,...,1)$$

$\downarrow$ **Project**

$$(xw/w,yw/w,zw/w,w,r/w,g/w,b,a/w,u/w,v/w,....,1/w)$$

$\downarrow$ **Interpolate**

Color - Gouraud ...

Both already approximations $\downarrow$ **Project**

...

Implicit Texture Transformation

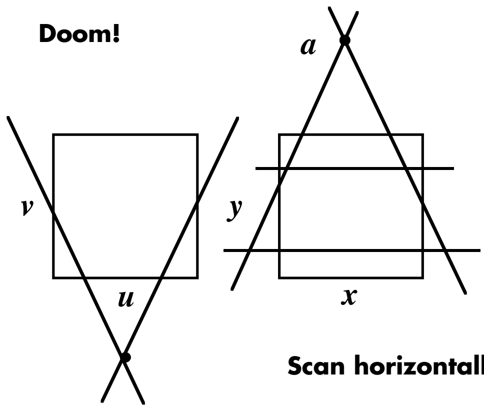
The Basic Idea

1. Projective transformation between spaces
2. Create homogeneous coordinates at vertices
3. Interpolate homogeneous coordinates
4. Divide through by w to get true coordinates

$$\begin{tikzcd} \hat{Q} \arrow[r, "T^{-1}"] & P \arrow[d] \\ Q \arrow[u] & \hat{P} \arrow[l, "T"] \end{tikzcd}$$

$$P = T^{-1}\hat{Q} = T^{-1}\frac{Q}{Q_w} = T^{-1}T\frac{\hat{P}}{Q_w} = \frac{\hat{P}}{Q_w}$$

Horizontal and Vertical Polygons

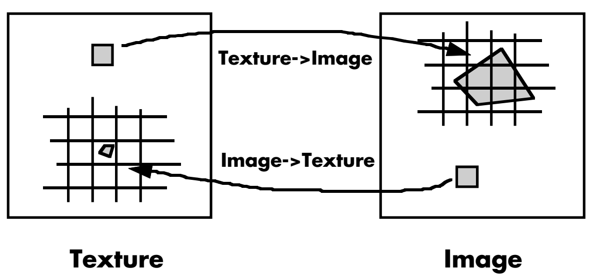


$$x = \frac{au}{a - v}$$

$$y = \frac{1}{a - v}$$

Scan horizontally: 1 divide per scanline

Filtering



Two cases:

1. **Magnification / Interpolation**
2. **Minification / Decimation / Averaging**

MipMaps