Transformations

```
gltranslate(2,2,2)
glrotate(45.,'z');

square();
```

What?

P′ = T(P)

Why?

Modeling

- Create objects in natural/convenient coordinates
- Multiple instances of a prototype shape
- Kinematics of linkages/skeletons - robotic anim

Viewing

- Virtual camera: parallel and perspective projections
- Windows and device independence

Types of Transformations

Continuous (preserves neighborhoods)

One to one, invertible

Classify by invariants or symmetries

- Isometry (distance preserved)
  - Reflections (interchanges left-handed and right-handed)
  - Rigid body motion: Rotations + Translations
- Similarity (preserves angles)
  - Uniform scale
- Affine (preserves parallel lines)
  - Non-uniform scales, shears or skews
- Collineation (lines remain lines)
  - Perspective
- Non-linear (lines become curves)
  - Twists, bends, warps, morphs, ...

Linear Transformations = Matrices

$$T(t_1P_1 + t_2P_2) = t_1T(P_1) + t_2T(P_2)$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} t_{xx} & t_{xy} & t_{xz} & t_{xw} \\ t_{yx} & t_{yy} & t_{yz} & t_{yw} \\ t_{zx} & t_{zy} & t_{zz} & t_{zw} \\ t_{wx} & t_{wy} & t_{wz} & t_{ww} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Translations

$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$

$$T(t_x, t_y, t_z) = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} x + t_x \\ y + t_y \\ z + t_z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Note: translations can be represented as matrices because of w!

What happens if w != 1?
What happens if w == 0?

Properties of Translations

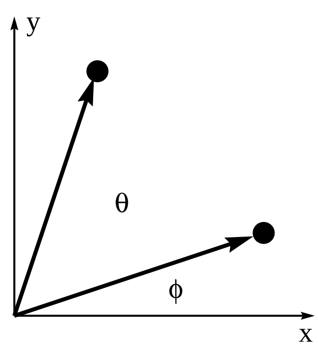
$$T(0,0,0) = I$$

$$T(s_x, s_y, s_z)T(t_x, t_y, t_z) = T(s_x + t_x, s_y + t_y, s_z + t_z)$$

$$T(s_x, s_y, s_z)T(t_x, t_y, t_z) = T(t_x, t_y, t_z)T(s_x, s_y, s_z)$$

$$T^{-1}(t_x, t_y, t_z) = T(-t_x, -t_y, -t_z)$$

Rotations



$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

$$x = r\cos\phi$$
$$y = r\sin\phi$$

$$x' = r\cos(\phi + \theta)$$
$$y' = r\sin(\phi + \theta)$$

$$\cos(\phi + \theta) = \cos\phi\cos\theta - \sin\phi\sin\theta$$
$$\sin(\phi + \theta) = \cos\phi\sin\theta + \sin\phi\cos\theta$$

$$x' = (r\cos\phi)\cos\theta - (r\sin\phi)\sin\theta$$
$$y' = (r\cos\phi)\sin\theta + (r\sin\phi)\cos\theta$$

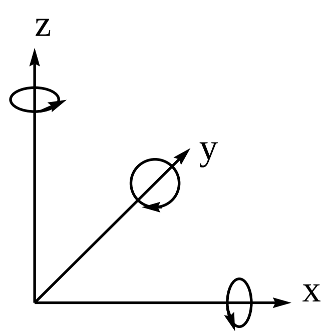
Rotation Matrices

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Right-Handed Rule



Right-handed in a right-handed coordinate system
Left-handed in a left-handed coordinate system!

Properties of Rotations

$$R_a(0) = I$$
$$R_a(\theta)R_a(\phi) = R_a(\theta+\phi)$$
$$R_a(\theta)R_a(\phi) = R_a(\phi)R_a(\theta)$$
$$R_a^{-1}(\theta) = R_a(-\theta) = R_a^T(\theta)$$

But,

$$R_a(\theta)R_b(\phi) \neq R_b(\phi)R_a(\theta)$$

Scale

$$x' = ax$$
$$y' = by$$
$$z' = cz$$

$$S(a,b,c) = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Other Types of Transformations

1. Rotations about an arbitrary axis

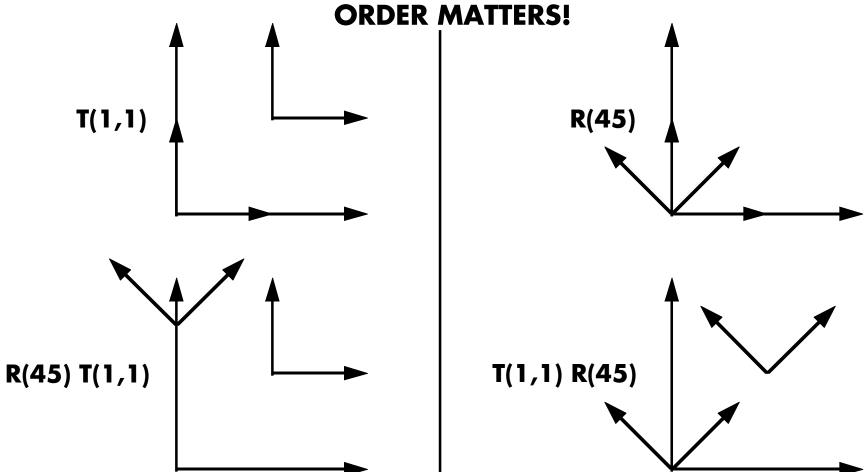
2. Reflections about an arbitrary plane

3. Shear along a plane

4. Perspective and orthographic projections

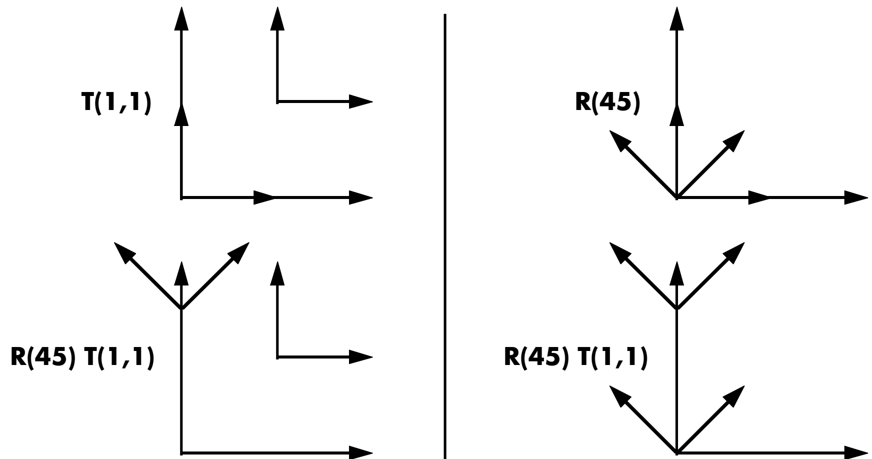
Combining a Translation and Rotation

ORDER MATTERS!



Transformations as Coordinate Systems

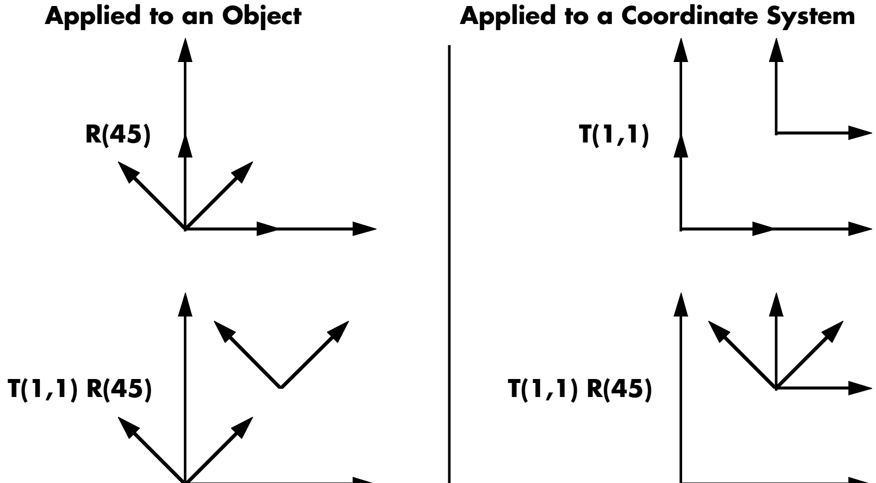
Applied to an Object | **Applied to a Coordinate System**



CS248 Lecture 11

Transformations as Coordinate Systems

Applied to an Object | **Applied to a Coordinate System**



CS248 Lecture 11

Transformations as Coordinate Systems

1. **Applying a transformation is equivalent to converting points from one coordinate system to another coordinate system.**

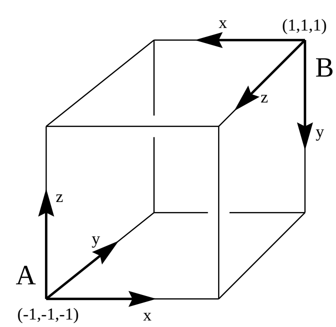
$$\boxed{{}_{B}T_{A}}$$

2. **The transformation in the other direction is the inverse**

$$\boxed{{}_{A}T_{B} = {}_{B}T_{A}^{-1}}$$

3. **The columns of a transformation matrix may be interpreted as the x, y, z, and origin of the axes defining a coordinate system**

Transformation Matrices = Axes



$$ {}_{C}T_{A} = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}_{A}T_{B} = \begin{pmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$