Viewing Transformations

Viewing coordinate systems

Planar geometric projections

- **Orthographic**
- **Perpspective**
  - **Perspective depth**
  - **Vanishing points and lines**
  - **Binocular stereo**
- **Axonometric**
- **Oblique**

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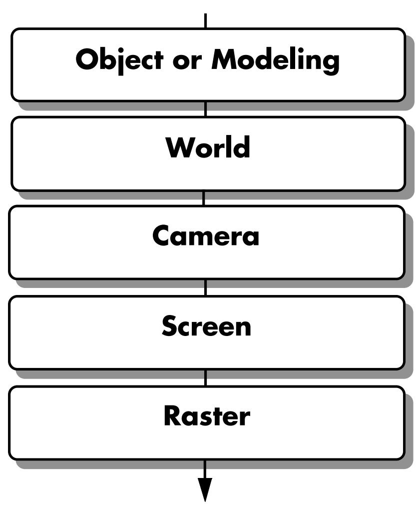
Viewing Coordinate Systems



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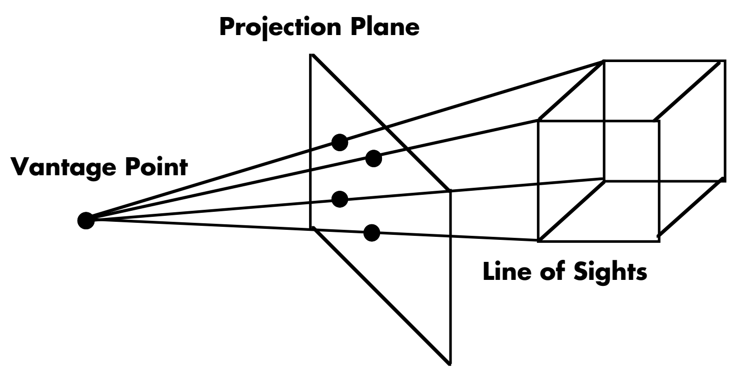
Planar Geometric Projection



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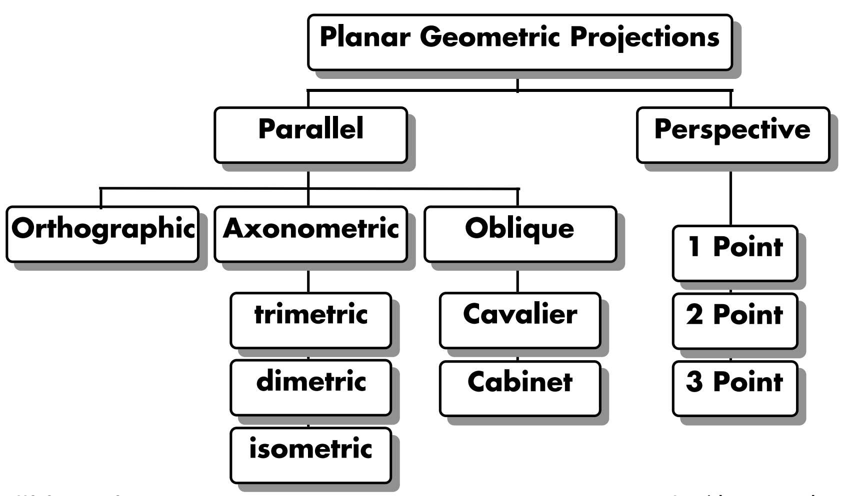
Types of Planar Geometric Projections



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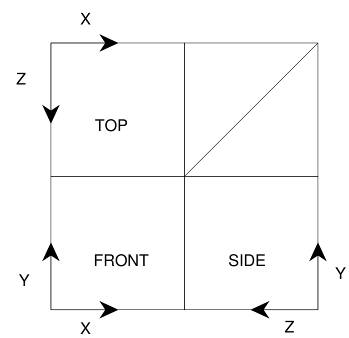
Orthographic Projections



Px=S(0,1,1)
Py=S(1,0,0)
Pz=S(1,1,0)

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Orthographic Transformation

```
glOrtho(float l, r, b, t, n, f);
```

Transforms to the unit cube: objects clipped to the cube

z's preserved for hidden surface calculations

Converts RHCS to LHCS: n and f relative

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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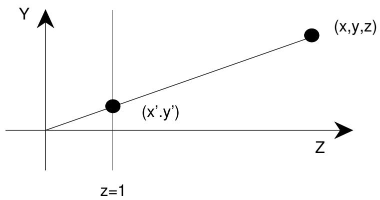
Perspective Projection



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ * & * & * & * \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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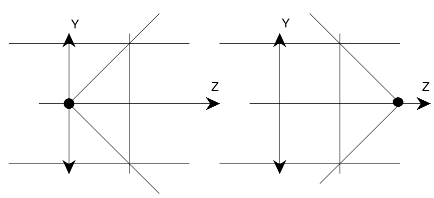
Perspective Depth

How should z transform?

$$z' = \frac{az + bw}{cz + dw}$$



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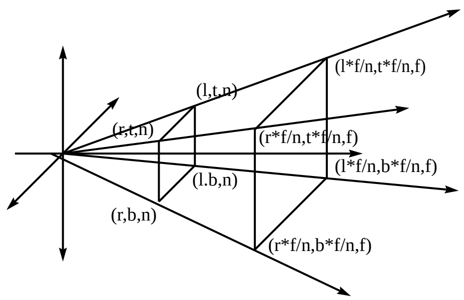
Perspective Frustum



$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

```
glFrustum(float l, r, b, t, n, f);
```

$$\lim_{f\to\infty} -\frac{f+n}{f-n} = -1$$

$$\lim_{f\to\infty} -\frac{2fn}{f-n} = -2n$$

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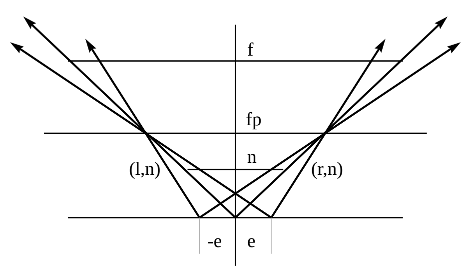
Binocular Stereo




```
stereo(float l,r,b,t,n,f, e, fp)
{
  frustum(l-e*n/fp,r-e*n/fp,b,t,n,f);
  translate(-e,0,0);
}
```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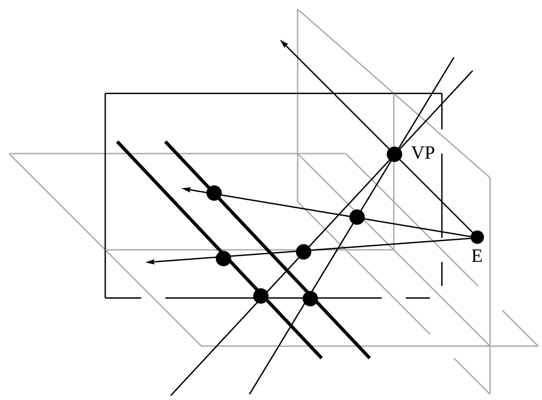
Vanishing Points and Lines



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"Synthetic Camera"

Move world relative to camera

- **Perform projection along the z-axis**

Move camera relative to world

- **Find the equivalent motion of the world ...**

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Axonometric Projection

Classify parallel projections by *foreshortening* of principal directions

- **trimetric: unequal**
- **dimetric: two equal**
- **isometric: three equal**

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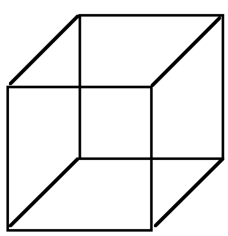
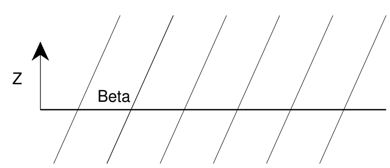
Oblique Projection





Cavalier

$$x' = x + z$$
$$y' = y + z$$

Cabinet

$$x' = x + \frac{1}{2}z$$
$$y' = y + \frac{1}{2}z$$

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Planar Projection Matrix

$$E = (x, y, z, w)$$

$$A = (a, b, c, d)$$

$$P(E, A) = (A \bullet E)I - EA^T$$

$$\begin{bmatrix} by + cz + dw & -bx & -cx & -dx \\ -ay & ax + cz + dw & -cy & -dy \\ -az & -bz & ax + by + dw & -dz \\ -aw & -bw & -cw & ax + by + cz \end{bmatrix}$$

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