# Intensity

**Intensity curves**

- **Stimulus/Response - Perception**
- **Input/Output - Gamma**

**Quantization**

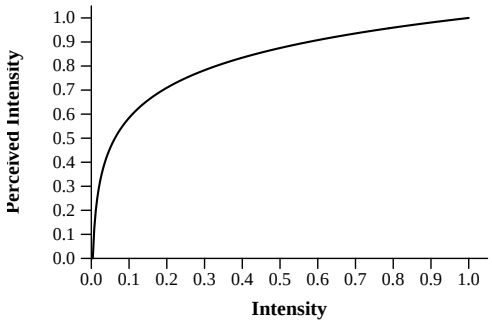
- **Uniform quantization**
- **Non-uniform quantization**

**Dithering and halftoning**

- **Random dither**
- **Ordered dither**
- **Clustered dither or halftone**
- **Error diffusion**


Copyright © Pat Hanrahan

# Perception of Intensities

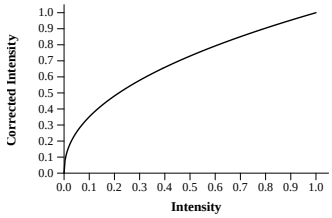


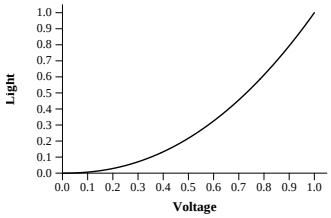
$$\Delta PI = \frac{\Delta I}{I} = \text{constant}$$

**Weber-Fechner Law**
**Logarithmic Response**


Copyright © Pat Hanrahan

# Gamma




**Camera $\gamma$=0.6** **Monitor $\gamma$=2.2**

$$L = V^{\gamma}$$


Copyright © Pat Hanrahan

# Gamma Correction

**Goal: Operate in linear light intensity space**

| Input | Output |
|---|---|
| 0 | 0 |
| ... | 0 |
| 11 | 0 |
| 12 | 1 |
| ... | 1 |
| 19 | 1 |
| ... | ... |
| 251 | 247 |
| 252 | 249 |
| 253 | 251 |
| 254 | 253 |
| 255 | 255 |

$$L = \left(V^{1/\gamma}\right)^{\gamma} = V$$

**Vidicon ($\gamma$=0.5) to Monitor ($\gamma$=2.8) = System $\gamma$=1.4**

**Graphics system: $\gamma$=1.2 or 1.7 or 2.4**


Copyright © Pat Hanrahan

## Quantization

**Definition: The process of turning a high precision number into a low precision number**

**Why?**

- **Conserve memory**
- **Binary output devices**

**How?**

- **Uniform perceptual steps**
- **Minimal error**

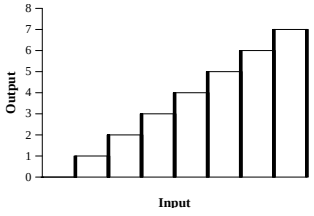

Copyright © Pat Hanrahan

## Uniform Quantization

**Uniform map:** $pv = trunc(K \times v)$

$0 \le v < 1$

$0 \le pv < K$




**K=256** **K=16** **K=4** **K=2**

**Notice contouring**


Copyright © Pat Hanrahan

# Quantization Error

**Quantization introduces error**

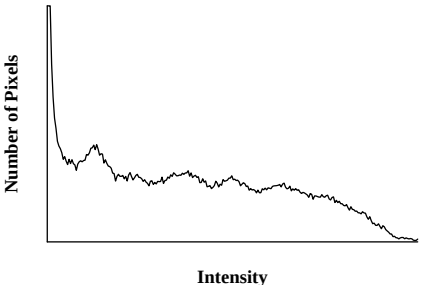
$$E^2 = \sum_{(x,y)} (pv(x,y) - K \times v(x,y))^2$$

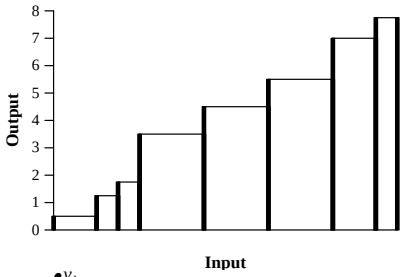
**To reduce error:**

1. **Optimal quantization (minimize error)**
2. **Dithering (trade-off intensity/space error)**


Copyright © Pat Hanrahan

# Non-uniform Quantization



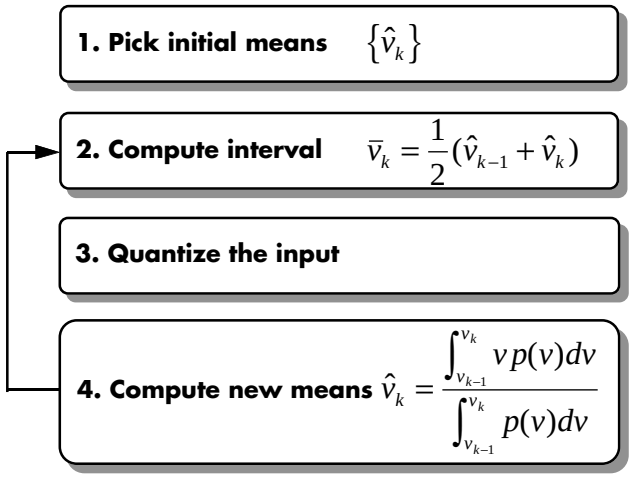

$$E^2 = \sum_{k=1}^{K} \int_{v_{k-1}}^{v_k} (v - \hat{v}_k)^2 p(v) dv$$

$$\hat{v}_k = \frac{\int_{v_{k-1}}^{v_k} vp(v)dv}{\int_{v_{k-1}}^{v_k} p(v)dv} \qquad \{v_k : 0 \le k < K\}$$

$$\bar{v}_k = \frac{1}{2}(\hat{v}_{k-1} + \hat{v}_k)$$


Copyright © Pat Hanrahan

## K-Means Algorithm





Copyright © Pat Hanrahan

## Dithering and Halftoning

**Trade spatial for intensity resolution**

- **Random dither; Robert's algorithm**
- **Ordered dither**
- **Clustered dither or halftones**
- **Error diffusion**

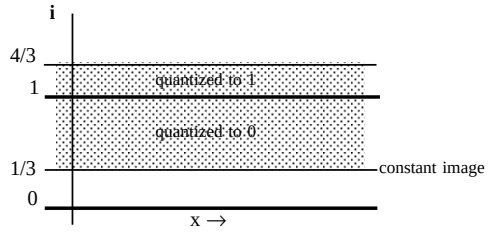
**Your eye will average over an area**


Copyright © Pat Hanrahan

## Robert's Algorithm

**First add noise** $\hat{v}(x,y) = trunc(K \times v(x,y) + noise(x,y))$

**Then quantize** $0 \le noise < 1$




**Moves low frequency (average error) to high frequency**

**Pink(low), *Blue (high)*, White(all) frequency noise**


Copyright © Pat Hanrahan

## Noise

**Noise functions**

- **Random function vs random sequence**
- **Some noise better than others: blue, ordered**
- **Reproducible and order independent**
- **Fast: precompute and store in a table**

```
for(y=0; y<ny; y++)
  for(x=0; x<nx; x++)
    vq[x][y] =
      (v[x][y]<threshold[x%M][y%M]) ? 0 : 1;
```

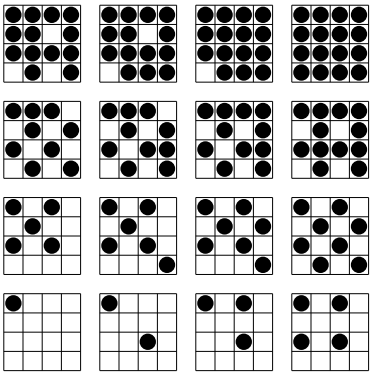

Copyright © Pat Hanrahan

## Bayer Ordered Dither Patterns

$$D_2 = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

$$D_n = \begin{bmatrix} 4D_{n/2} + 0 & 4D_{n/2} + 2 \\ 4D_{n/2} + 3 & 4D_{n/2} + 1 \end{bmatrix}$$

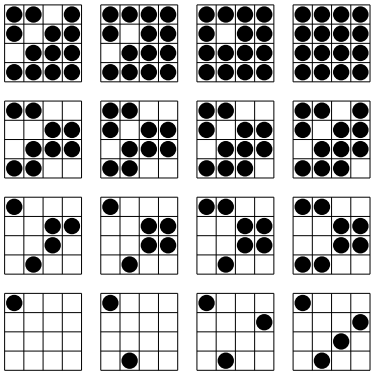
$$D_4 = \begin{bmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{bmatrix}$$



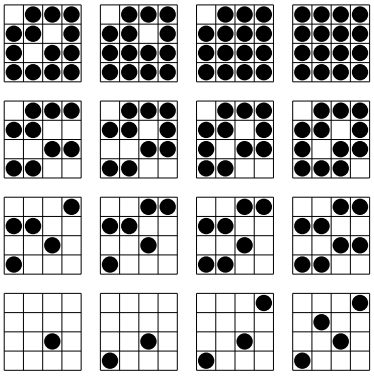


Copyright © Pat Hanrahan

## Magic Squares



$$M_4 = \begin{bmatrix} 0 & 6 & 13 & 11 \\ 9 & 15 & 4 & 2 \\ 14 & 8 & 3 & 5 \\ 7 & 1 & 10 & 12 \end{bmatrix}$$



$$P_4 = \begin{bmatrix} 15 & 8 & 5 & 2 \\ 4 & 3 & 14 & 9 \\ 10 & 13 & 0 & 7 \\ 1 & 6 & 11 & 12 \end{bmatrix}$$



Copyright © Pat Hanrahan

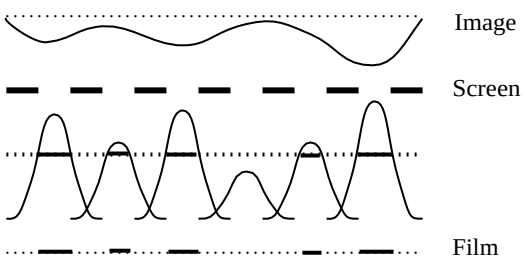
## Halftoning

**Printers: *Binary Blobs* of ink**

- **Clustered pattern**
- **Enlarging spot**

**Classical printer screens**

- **Spot function**
- **Line frequency**
- **Tilt angle**



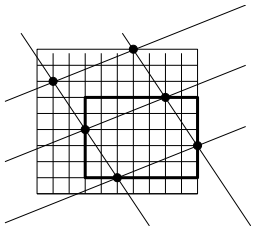


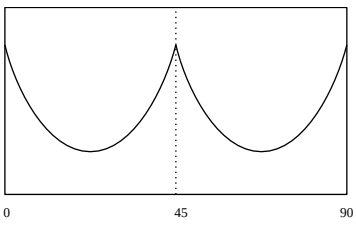
Copyright © Pat Hanrahan

## Screen Patterns

**Interplay between discrete angles and discrete spacing**

- **Given printer dpi implies**
  - **High spatial resolution means low intensity resolution**
- **Angle must be ratios of integers m/n**
  - **45 works best; different angles for colors, moires**







Copyright © Pat Hanrahan

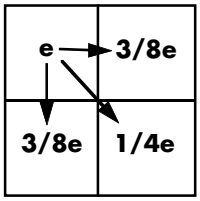
# Error Diffusion

**Idea: Quantize, then distribute error to neighbors**

```
for(y=0; y<ny; y++)
  for(x=0; x<nx; x++){
    vq[x][y] = quantize(v[x][y]);

    e = v[x][y] - vq[x][y];

    i[x+1][y] += 3/8*e;
    i[x][y+1] += 3/8*e;
    i[x+1][y+1] += 1/4*e;
}
```





Copyright © Pat Hanrahan