Signal Processing and Sampling

The sampling and reconstruction process

- Real world: continuous
- Digital world: discrete

The Fourier Transform and spectra

Filtering

The convolution theorem

The sampling theorem

Aliasing and antialiasing

Why? Scan conv., aliasing artifacts, generally useful, EE …

Image Capture

Physical cameras convert a continuous image to a discrete sampled image by integrating over the active area of a sensor.

$$R = \int_A E(x,y)\,P(x,y)\,dx\,dy$$

Examples:

- Vidicon: phosphors
- CCD array
- Retina: photoreceptors

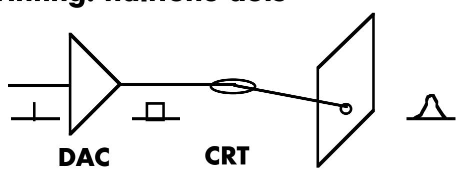
Virtual "computer graphics" cameras do not integrate, instead they simply sample …

Image Generation

Physical displays recreate a continuous image from a discrete sampled image by using a finite sized source of light

Examples:

- **Cathode ray tube: phosphors**
- **DAC's: sample and hold**
- **Printing: halftone dots**



Sampling in Computer Graphics

Artifacts due to sampling - Aliasing

- **Jaggies**
- **Moire**
- **Flickering small objects**
- **Sparkling highlights**
- **Temporal strobing**

Preventing these artifacts - Antialiasing

First computer graphics commandment:

Thou shalt not make pictures with computational artifacts

Spectral Analysis / Fourier Transforms

Each function has two representations

- **Spatial (time) domain - normal representation**
- **Frequency domain - spectral representation**

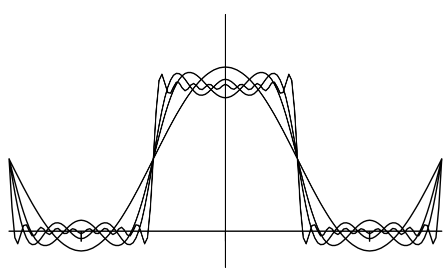
Spectral representation treats the function as a sum of sines and cosines

The *Fourier transform* converts between the spatial and frequency domain

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$$

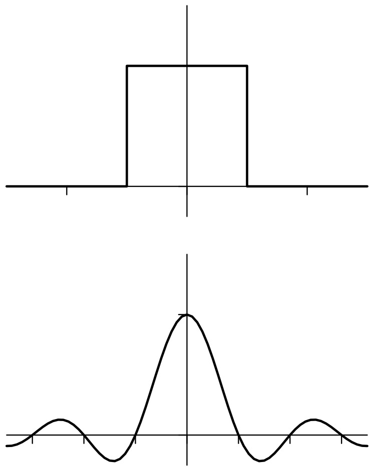
$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{i\omega x}\, d\omega$$

Example: Fourier Series of a Pulse



$$S_n(x) = \frac{1}{2} + \frac{2}{\pi}(\cos \omega x - \frac{1}{3}\cos 3\omega x + \frac{1}{5}\cos 5\omega x + \ldots)$$
$$= \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{n}(-1)^k \frac{\cos(2k-1)\omega x}{2k-1}$$

The "Sinc" Function



$$\int_{-\infty}^{\infty} square(x) e^{-iwx}\, dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-iwx}\, dx$$

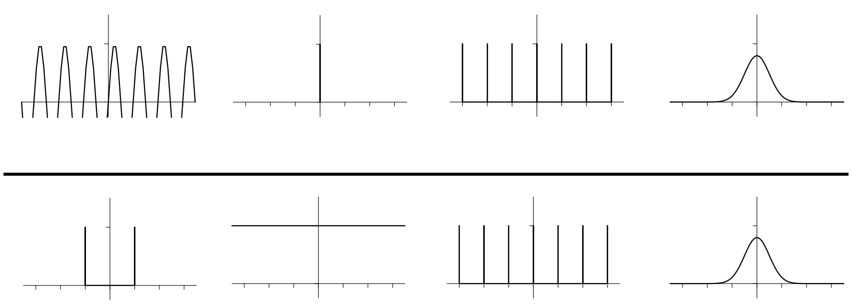
$$= \left. \frac{e^{-iwx}}{-iwx} \right|_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= \frac{e^{-\frac{1}{2}iw} - e^{\frac{1}{2}iw}}{\frac{1}{2}iw + \frac{1}{2}iw}$$

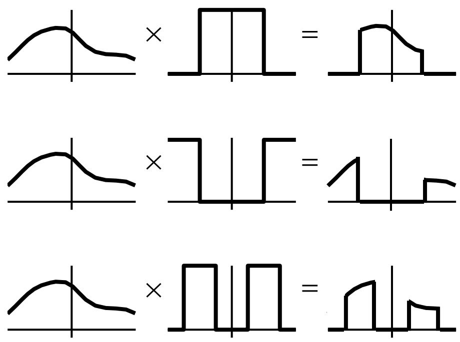
$$= \frac{\sin \frac{1}{2} w}{\frac{1}{2} w}$$

$$= \operatorname{sinc} f$$

Fourier Transform Pairs

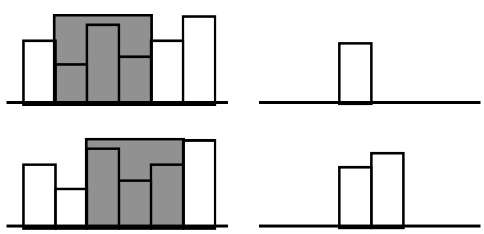


Filtering: Frequency Domain



Filtering: Spatial Domain

Filtering



Convolution of two functions

$$h(x) = f \otimes g = \int f(y) g(x-y)\, dy$$

Convolution Theorem

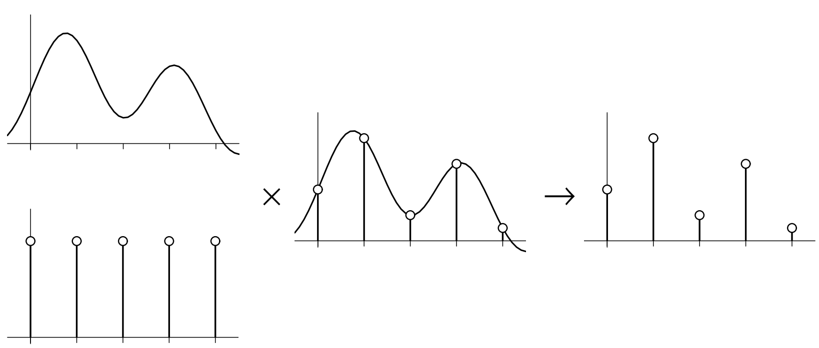
***Theorem*: Multiplication in the frequency domain is equivalent to convolution in the space domain.**

$$F \times G \quad \Leftrightarrow \quad f \otimes g$$

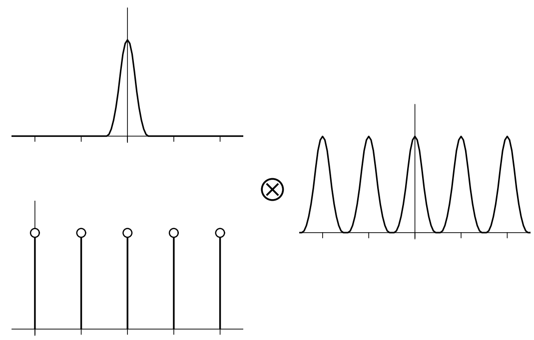
***Symmetric Theorem*: Multiplication in the space domain is equivalent to convolution in the frequency domain.**

$$f \times g \quad \Leftrightarrow \quad F \otimes G$$

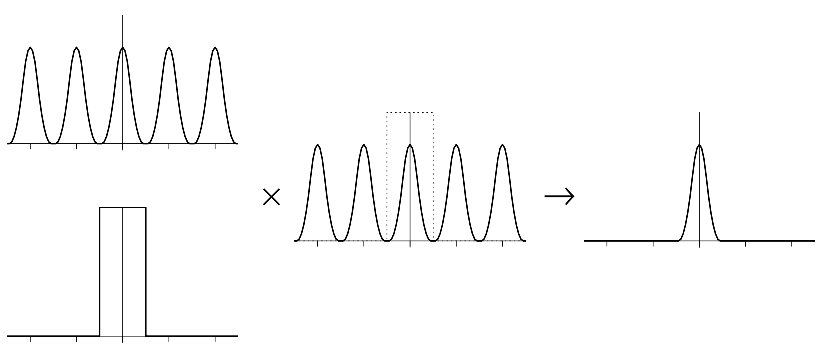
Sampling: Spatial Domain



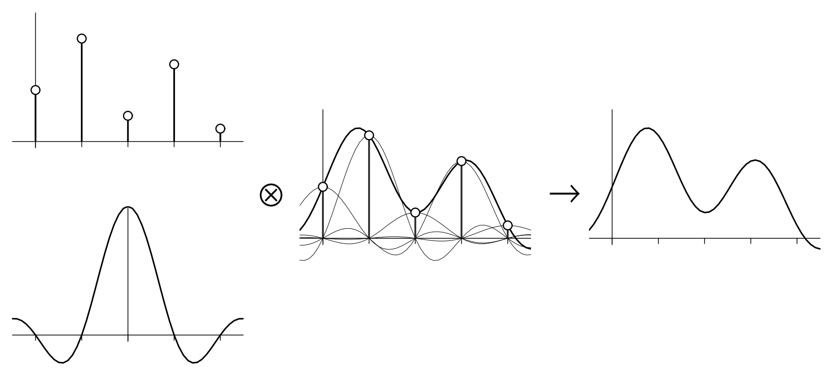
Sampling: Frequency Domain



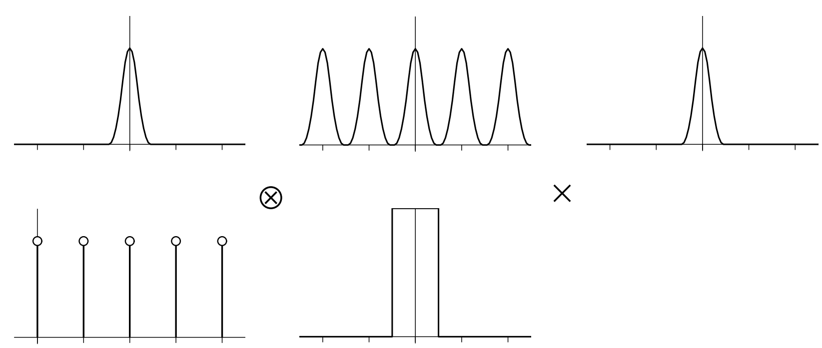
Reconstruction: Frequency Domain



Reconstruction: Spatial Domain



Sampling and Reconstruction



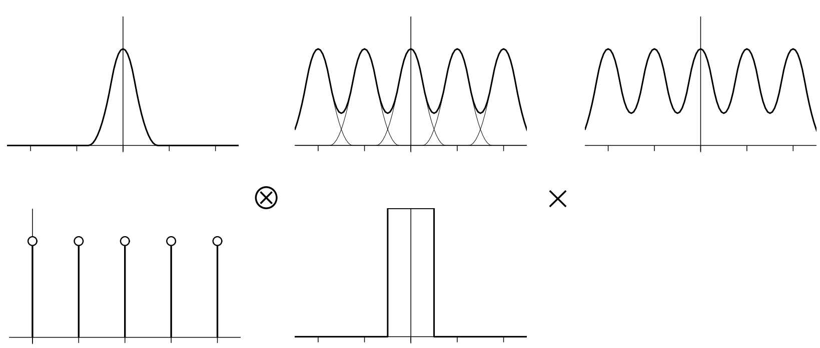
Sampling Theorem

This result if known as the *Sampling Theorem* and is due to Claude Shannon who first discovered it in 1949

> ***A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above 1/2 the sampling frequency***

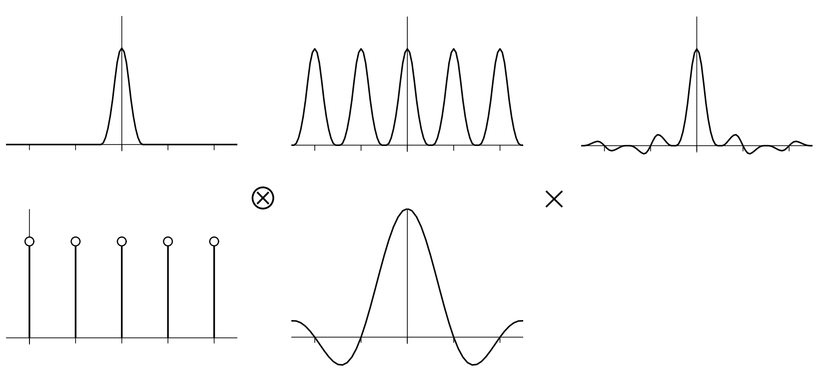
For a given bandlimited function, the rate at which it must be sampled is called the *Nyquist Frequency*

Undersampling: Pre-Aliasing

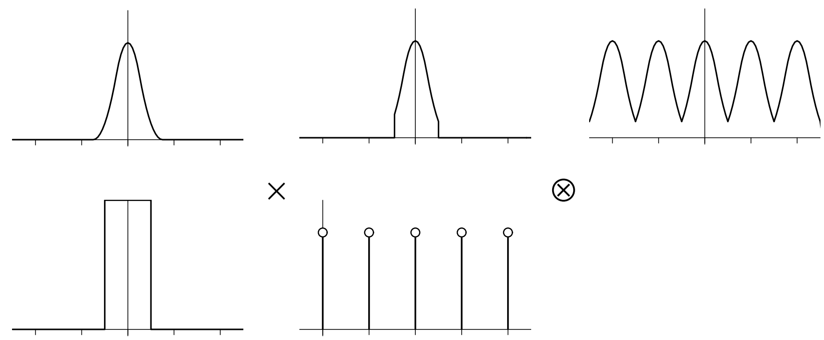


“Aliases”

Poor reconstruction: Post-Aliasing



Antialiasing by Prefiltering



Antialiasing by Prefiltering

Ideally: convolve with a sinc function to bandlimit the function at the Nyquist sampling rate.

Typically: convolve with a box filter (area average) before sampling

Practically:

- **Constant colored polygonal fragments doable**
- **Complex environments not doable**

Recall that physical cameras integrates over an sensor area