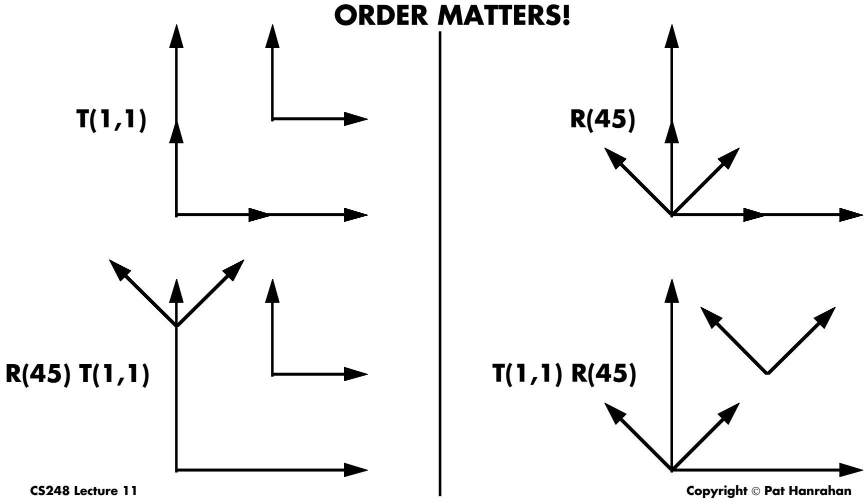
Combining a Translation and Rotation

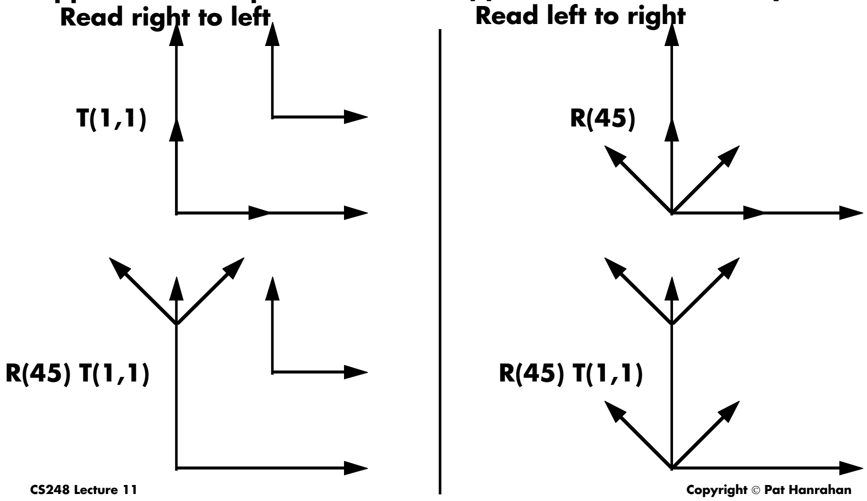
ORDER MATTERS!



Transformations as Coordinate Systems

Applied to an Object
Read right to left

Applied to a Coordinate System
Read left to right



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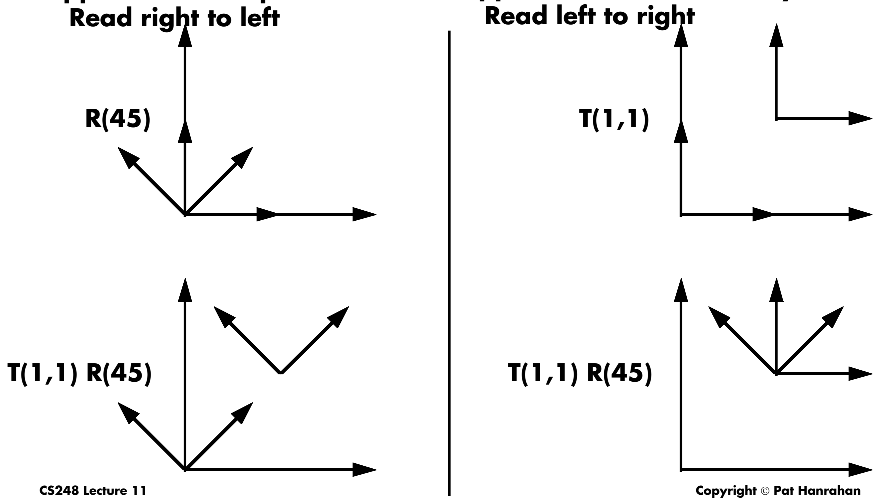
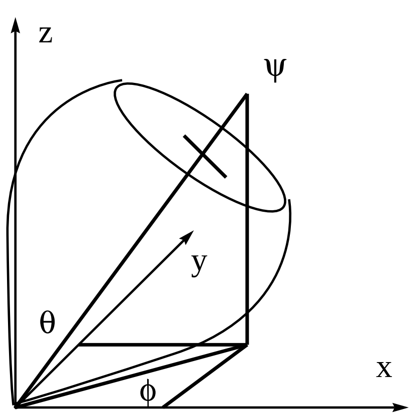


Combining Rotations: Euler Angles



Transformations as Coordinate Systems

1. Applying a transformation is equivalent to converting points from one coordinate system to another coordinate system.

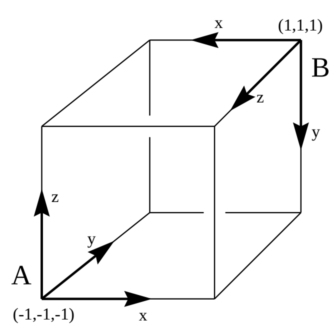
$$\boxed{{}_{B}T_{A}}$$

2. The transformation in the other direction is the inverse

$$\boxed{{}_{A}T_{B} = {}_{B}T_{A}^{-1}}$$

3. The columns of a transformation matrix may be interpreted as the x, y, z, and origin of the axes defining a coordinate system

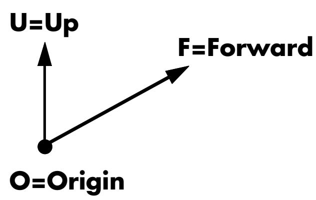
Transformation Matrices = Axes

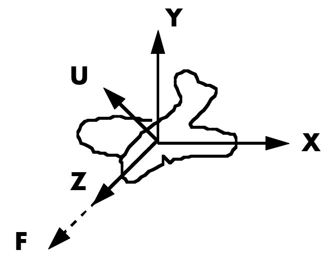


$${}_{W}T_{A} = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}_{A}T_{B} = \begin{pmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Orientation Matrix





W = O

Z = F

X = U x Z

Y = Z x X

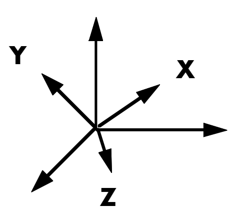
T=(X Y Z W)

Rotation Matrix = Orthogonal Matrix

$$R^T R = \begin{pmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{pmatrix} \begin{pmatrix} \hat{X} & \hat{Y} & \hat{Z} \end{pmatrix} = \begin{pmatrix} \hat{X}\bullet\hat{X} & \hat{X}\bullet\hat{Y} & \hat{X}\bullet\hat{Z} \\ \hat{Y}\bullet\hat{X} & \hat{Y}\bullet\hat{Y} & \hat{Y}\bullet\hat{Z} \\ \hat{Z}\bullet\hat{X} & \hat{Z}\bullet\hat{Y} & \hat{Z}\bullet\hat{Z} \end{pmatrix}$$

$$R = \begin{pmatrix} \hat{X} & \hat{Y} & \hat{Z} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$



$$\boxed{R^{-1} = R^T}$$

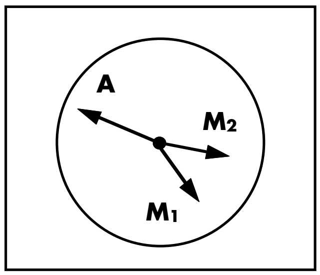
Rolling Ball

Control rotation by mouse difficult

mouse x -> pan

mouse y -> tilt

Rolling Ball metaphor



$$\vec{M} = \begin{pmatrix} x & y & \sqrt{1 - x^2 - y^2} \end{pmatrix}$$

$$\vec{A} = \vec{M}_1 \times \vec{M}_2$$

$$R_1 = \begin{pmatrix} \hat{M}_1 & \hat{A} \times \hat{M}_1 & \hat{A} \end{pmatrix}$$

$$R_2 = \begin{pmatrix} \hat{M}_2 & \hat{A} \times \hat{M}_2 & \hat{A} \end{pmatrix}$$

$$R = R_2 R_1^T$$

Composition of Transformations

- **Sequence of transforms = single transform**

  P′ = A (B (C (D (P))))

  = (A B C D) P

  = T P

- **Inverse of a sequence**

  P = (D^{-1} C^{-1} B^{-1} A^{-1}) (A B C D) P

  Therefore

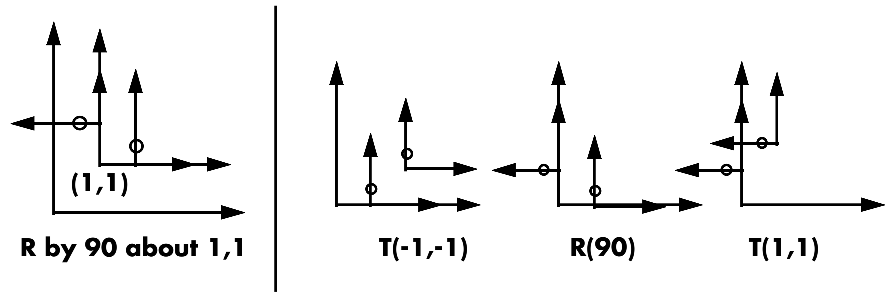
  (A B C D) $^{-1}$ = D^{-1} C^{-1} B^{-1} A^{-1}

  1. Apply inverses in the reverse order

  2. Concatenate and numerically invert matrix

Transforming Transformations

Apply a transformation in another coordinate system



More generally: $R_T = T\ R\ T^{-1}$

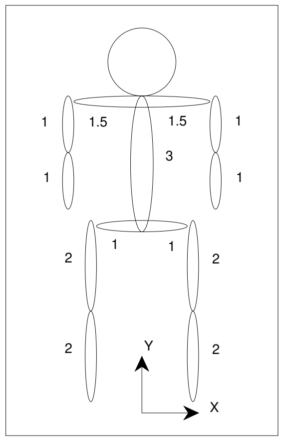
Current Transformation Matrix (CTM)

- **Graphics systems maintain a *current transformation matrix (CTM).* All geometry is transformed by the CTM.**
- **The CTM defines the current or *object* or *local* coordinate system. All geometry is defined in the current coordinate system.**
- **Transformation commands are concatenated onto the ctm. Note: The last transformation specified is the first to be performed.**

  CTM′ = CTM * T

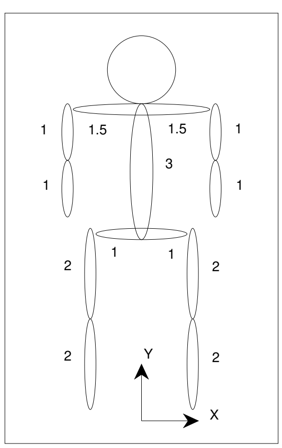
- **The CTM may be pushed onto a transformation stack, and latter popped to restore a previous state.**

Skeleton




```
body
  torso
    head
    shoulder
      larm
        upperarm
        lowerarm
        hand
      rarm
        upperarm
        lowerarm
        hand
    hips
      lleg
        upperleg
        lowerleg
        foot
      rleg
        upperleg
        lowerleg
        foot
```


Skeleton




```
translate(0,4,0)
torso();
pushmatrix();
    translate(0,3,0);
    shoulder();
    pushmatrix();
      rotatey(necky);
      rotatex(neckx);
      head();
    popmatrix();
    pushmatrix();
      translate(1.5,0,0);
      rotatex(lshoulderx);
      upperarm();
      pushmatrix();
        translate(0,-1,0);
        rotatex(lelbowx);
        lowerarm();
        ...
      popmatrix();
    popmatrix();
...
```