Viewing Transformations

Viewing coordinate systems

Planar geometric projections

- **Orthographic**
- **Perpspective**
  - **Perspective depth**
  - **Vanishing points and lines**
  - **Binocular stereo**
- **Axonometric**
- **Oblique**

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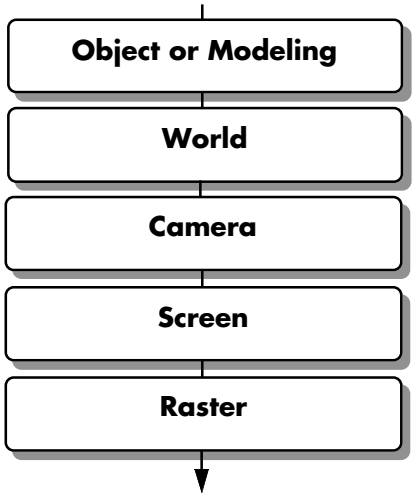
Viewing Coordinate Systems



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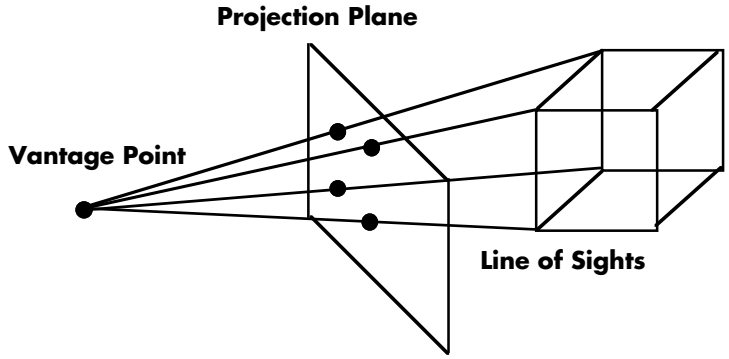
Planar Geometric Projection



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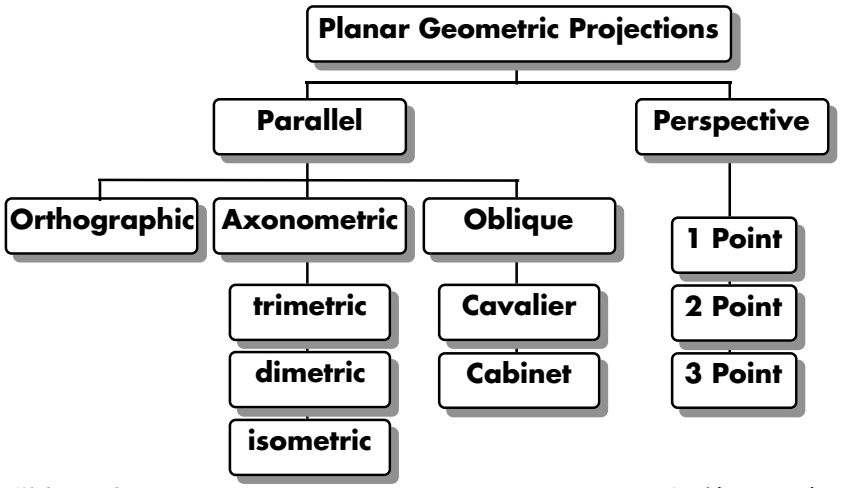
Types of Planar Geometric Projections



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"Synthetic Camera"

Cameras in GL

Fix camera, move world relative to camera

Always perform projection along the z-axis

```
gluLookat(ex,ey,ez,atx,aty,atz,upx,upy,upx);
```

Similar to the "airplane" example from last lecture

Better: Treat the camera like any other object

Then find the equivalent motion of the world ...

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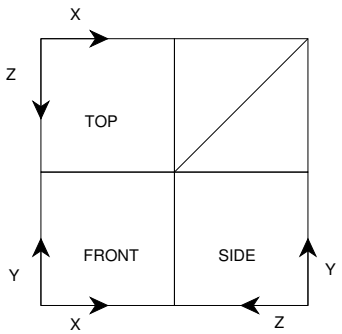
Orthographic Projections



Px=S(0,1,1)
Py=S(1,0,1)
Pz=S(1,1,0)

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Orthographic Transformation

`glOrtho(float l, r, b, t, n, f);`

Transforms to the unit cube: objects clipped to the cube

z's preserved for hidden surface calculations

Converts RHCS to LHCS: n and f relative

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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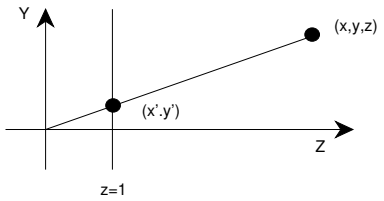
Perspective Projection



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ * & * & * & * \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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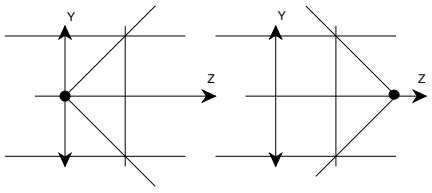
Perspective Depth

How should z transform?

$$z' = \frac{az + bw}{cz + dw}$$



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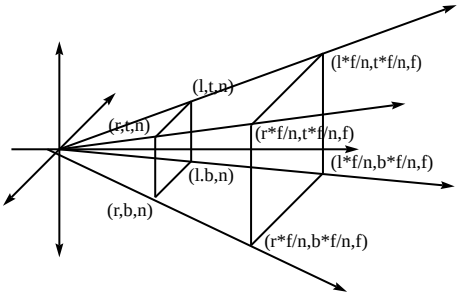
Perspective Frustum



$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

```
glFrustum(float l, r, b, t, n, f);
```

$$\lim_{f\to\infty} -\frac{f+n}{f-n} = -1$$

$$\lim_{f\to\infty} -\frac{2fn}{f-n} = -2n$$

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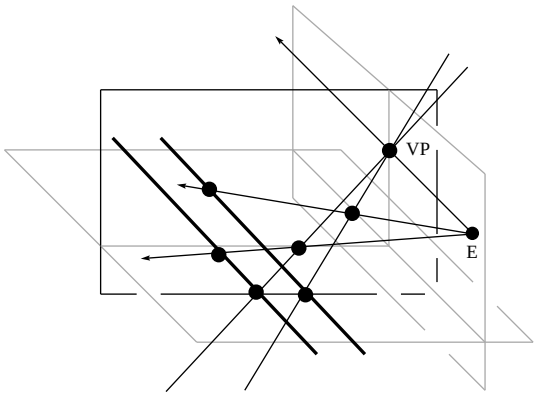
Vanishing Points and Lines



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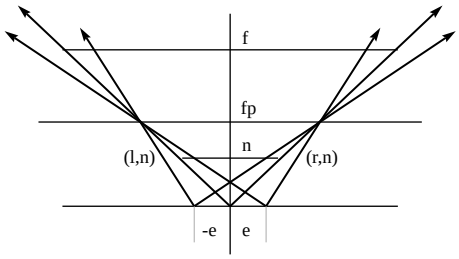
Binocular Stereo




```
stereo(float l,r,b,t,n,f, e, fp)
{
  frustum(l-e*n/fp,r-e*n/fp,b,t,n,f);
  translate(-e,0,0);
}
```

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Axonometric Projection

Classify parallel projections by *foreshortening* of principal directions

- **trimetric: unequal**
- **dimetric: two equal**
- **isometric: three equal**

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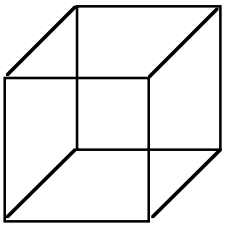
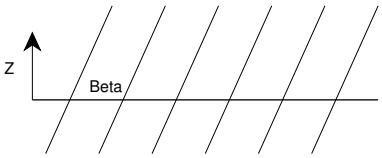
Oblique Projection





Cavalier

$$x' = x + z$$
$$y' = y + z$$

Cabinet

$$x' = x + \frac{1}{2}z$$
$$y' = y + \frac{1}{2}z$$

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