Illumination Algorithms

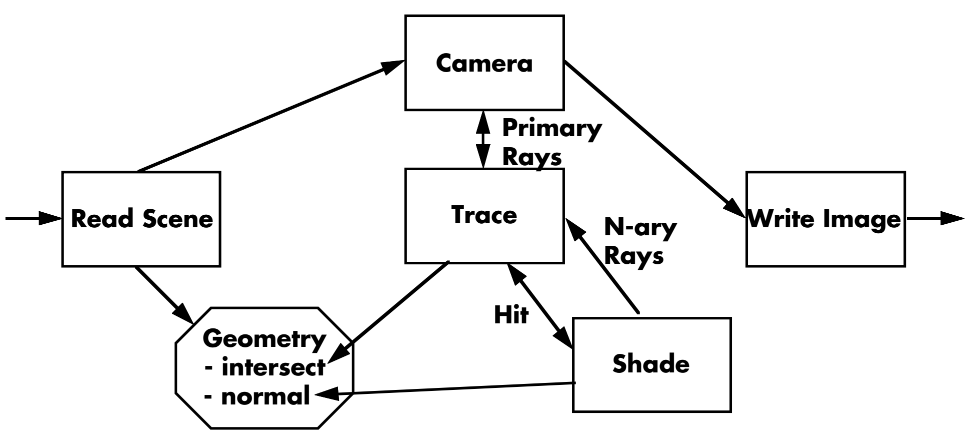
OpenGL

- Only point/distant light sources - not area
- Assumes objects are not "shadowed"
- Doesn't handle interreflection

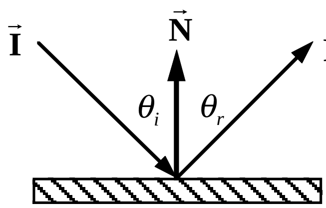
Ray Tracing

- Basic ray tracing algorithm
- Reflection and refraction
- Ray-object intersections
  - Plane
  - Polygon
  - Sphere
- Acceleration schemes

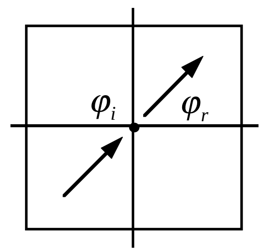
Classic Ray Tracing Algorithm



Law of Reflection



$$\theta_r = \theta_i$$

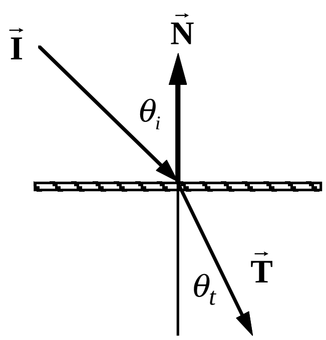


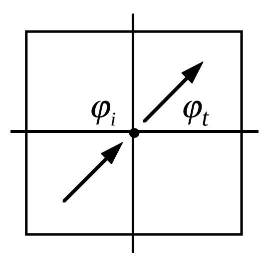
$$\varphi_r = \varphi_i \pm \pi$$

$$\vec{\mathbf{R}} + (-\vec{\mathbf{I}}) = 2\cos\theta\,\vec{\mathbf{N}} = -(\vec{\mathbf{I}} \bullet \vec{\mathbf{N}})\vec{\mathbf{N}}$$

$$\vec{\mathbf{R}} = \vec{\mathbf{I}} - (\vec{\mathbf{I}} \bullet \vec{\mathbf{N}})\vec{\mathbf{N}}$$

Snell's Law

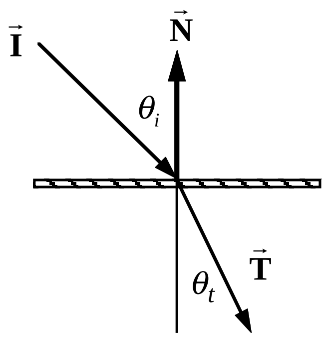




$$\varphi_t = \varphi_i \pm \pi$$

$$n_i \sin\theta_i = n_t \sin\theta_t \qquad n_i\,\vec{\mathbf{N}} \times \vec{\mathbf{T}} = n_t\,\vec{\mathbf{N}} \times \vec{\mathbf{I}}$$

Law of Refraction



$$\vec{\mathbf{N}} \times \vec{\mathbf{T}} = \mu \vec{\mathbf{N}} \times \vec{\mathbf{I}} \qquad \mu = n_i / n_t$$

$$\vec{\mathbf{N}} \times (\vec{\mathbf{T}} - \mu \vec{\mathbf{I}}) = 0 \qquad \vec{\mathbf{T}} = \mu \vec{\mathbf{I}} + \gamma \vec{\mathbf{N}}$$

$$\vec{\mathbf{T}}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \vec{\mathbf{I}} \bullet \vec{\mathbf{N}}$$

$$\gamma = -\mu \vec{\mathbf{I}} \bullet \vec{\mathbf{N}} \pm \left\{ 1 - \mu^2 (1 - \left(\vec{\mathbf{I}} \bullet \vec{\mathbf{N}}\right)^2) \right\}^{1/2}$$

$$= \mu \cos\theta_i \pm \left\{ 1 - \mu^2 \sin^2 \theta_i \right\}^{1/2}$$

$$= \mu \cos\theta_i \pm \cos\theta_t$$

$$= \mu \cos\theta_i - \cos\theta_t \qquad \leftarrow \gamma = \mu - 1$$

Total internal reflection:

$$1 - \mu^2 (1 - \left(\vec{\mathbf{I}} \bullet \vec{\mathbf{N}}\right)^2) < 0$$

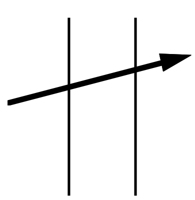
Ray-Plane Intersection

Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\vec{\mathbf{D}}$

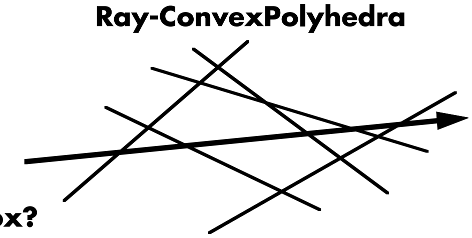
Plane: $\vec{\mathbf{P}} \bullet \vec{\mathbf{N}} + d = 0$

$$ax + by + cz + d = 0$$

Ray-Slab



Ray-ConvexPolyhedra



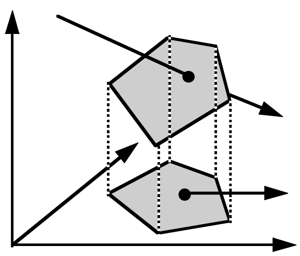
Ray-Box?

Ray-Zonahedra?

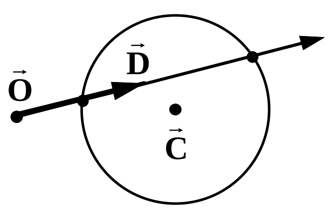
Copyright © Pat Hanrahan

Ray-Polygon Intersection

1. Find intersection with plane of support
2. Test whether point is inside 3D polygon
    a. Project onto xy plane (actually use max N coord.)
    b. Test whether point is inside 2D polygon



Ray-Sphere Intersection



Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\,\vec{\mathbf{D}}$

Sphere: $\left(\vec{\mathbf{P}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$

$$\left(\vec{\mathbf{O}} - t\,\vec{\mathbf{D}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$$

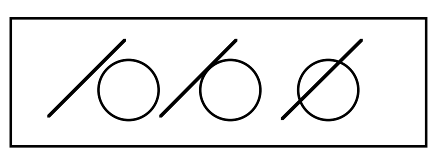
$$at^2 + bt + c = 0$$

$$a = \vec{\mathbf{D}}^2 = 1$$

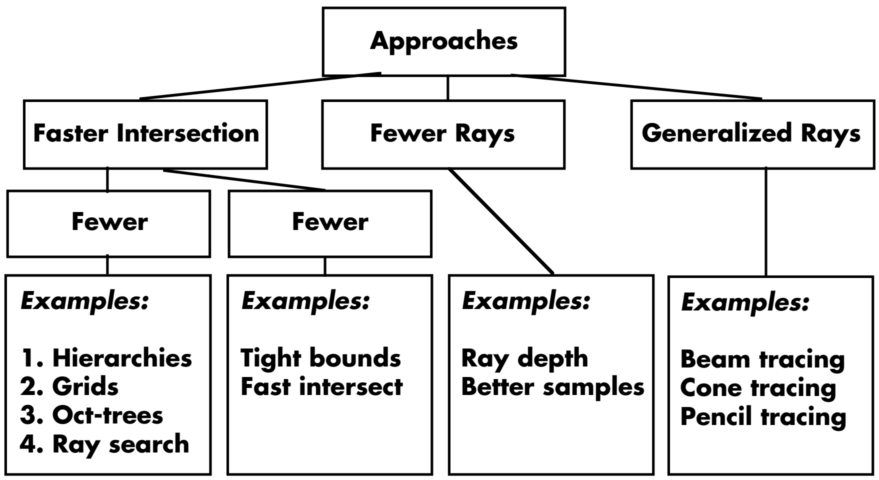
$$b = 2(\vec{\mathbf{O}} - \vec{\mathbf{C}}) \bullet \vec{\mathbf{D}}$$

$$c = (\vec{\mathbf{O}} - \vec{\mathbf{C}})^2 - R^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Ray Tracing Acceleration Techniques



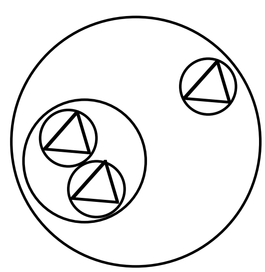
Hierarchical Bounding Volumes

Create of tree of bounding volumes

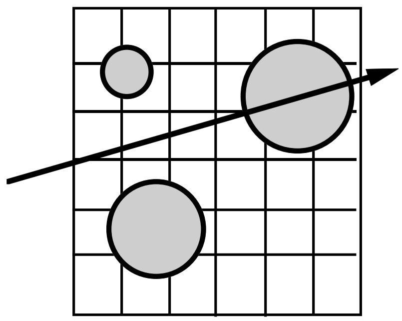
- **Children are contained within parent**
- **Creation preprocess**
  - **From model hierarchy**
  - **Automatically**

Search

```
Test parent
if intersect
   recurse on children
     in ray order
```



Grids



Fujimoto *et al.*

1. Preprocess scene
   Place object in cell if overlaps cell
2. Traverse grid
   3D line drawing

What resolution?