Signal Processing and Sampling

The sampling and reconstruction process

- Real world: continuous
- Digital world: discrete

The Fourier Transform and spectra

Filtering

The convolution theorem

The sampling theorem

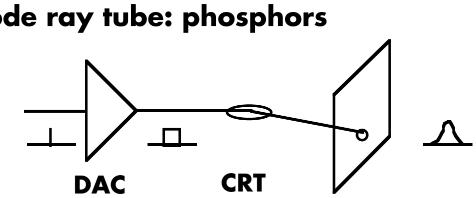
Aliasing and antialiasing

Displays = Signal Reconstruction

All physical displays recreate a continuous image from a discrete sampled image by using a finite sized source of light for each pixel.

Examples:

- DAC's: sample and hold
- Cathode ray tube: phosphors



Imagers = Signal Sampling

All digital cameras convert a continuous image to a discrete sampled image by integrating over the active area of a sensor.

$$R(i,j) = \int_{A_{i,j}} E(x,y)\, P_{i,j}(x,y)\, dx\, dy$$

Examples:

- Retina: photoreceptors
- CCD array
- Vidicon: phosphors

Virtual "computer graphics" cameras do not integrate, instead they simply sample ...

Sampling in Computer Graphics

Artifacts due to sampling - Aliasing

- Jaggies
- Moire
- Flickering small objects
- Sparkling highlights
- Temporal strobing

Preventing these artifacts - Antialiasing

First computer graphics commandment:

Thou shalt not make pictures with computational artifacts

Spectral Analysis / Fourier Transforms

Spectral representation treats the function as a weighted sum of sines and cosines

Each function has two representations

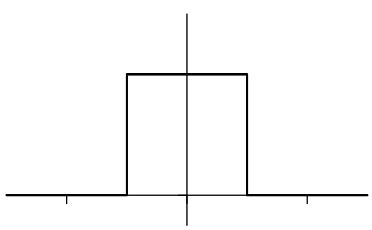
- **Spatial (time) domain - normal representation**
- **Frequency domain - spectral representation**

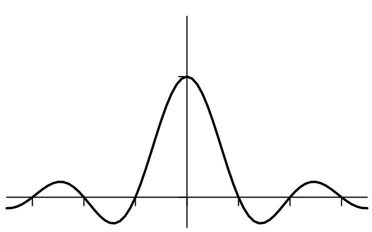
The *Fourier transform* converts between the spatial and frequency domain

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega x}\, d\omega$$

The "Sinc" Function





$$\int_{-\infty}^{\infty} square(x) e^{-iwx}\, dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-iwx}\, dx$$

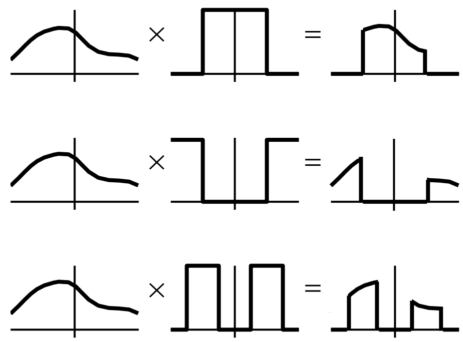
$$= \left. \frac{e^{-iwx}}{-iwx} \right|_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= \frac{e^{-\frac{1}{2}iw} - e^{\frac{1}{2}iw}}{\frac{1}{2}iw + \frac{1}{2}iw}$$

$$= \frac{\sin \frac{1}{2} w}{\frac{1}{2} w}$$

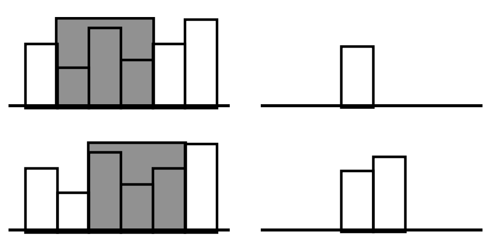
$$= \operatorname{sinc} f$$

Filtering: Frequency Domain



Filtering: Spatial Domain

Filtering



Convolution of two functions

$$h(x) = f \otimes g = \int f(y)g(x-y)\,dy$$

Convolution Theorem

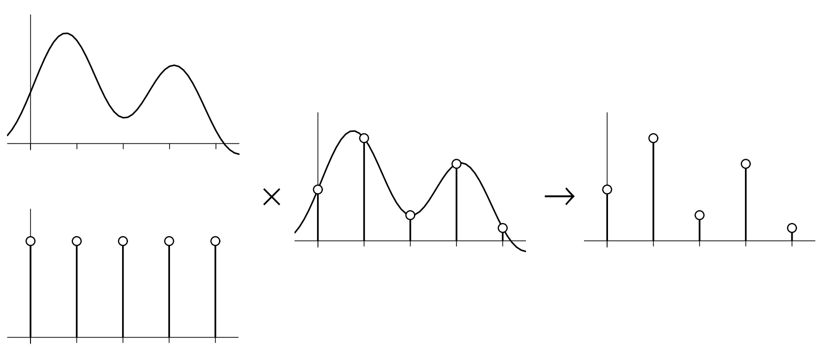
***Theorem*: Multiplication in the frequency domain is equivalent to convolution in the space domain.**

$$F \times G \quad \Leftrightarrow \quad f \otimes g$$

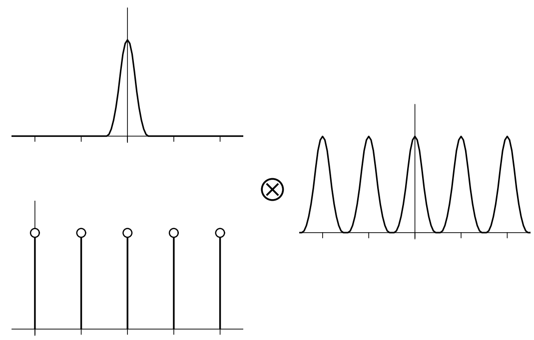
***Symmetric Theorem*: Multiplication in the space domain is equivalent to convolution in the frequency domain.**

$$f \times g \quad \Leftrightarrow \quad F \otimes G$$

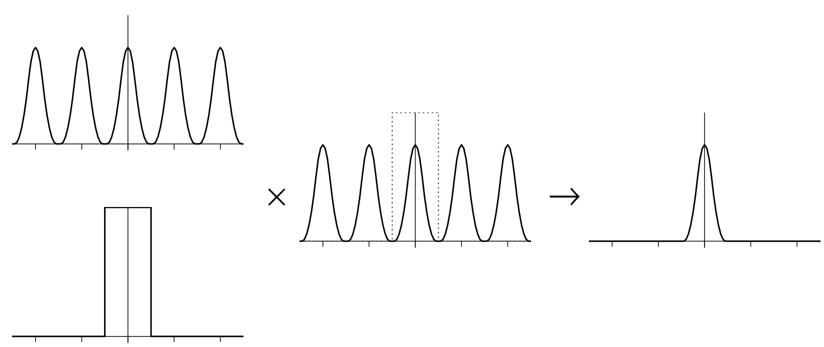
Sampling: Spatial Domain



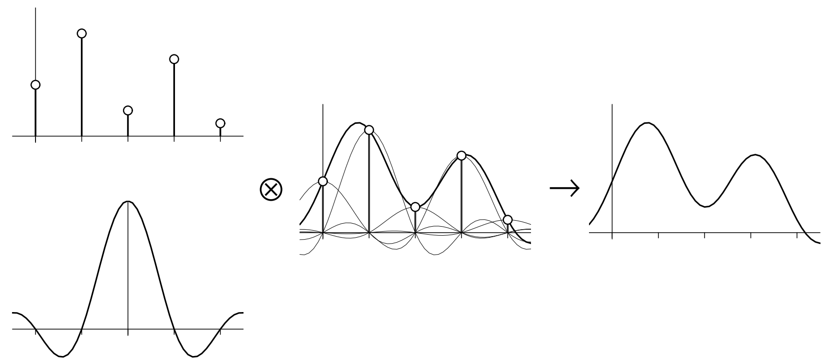
Sampling: Frequency Domain



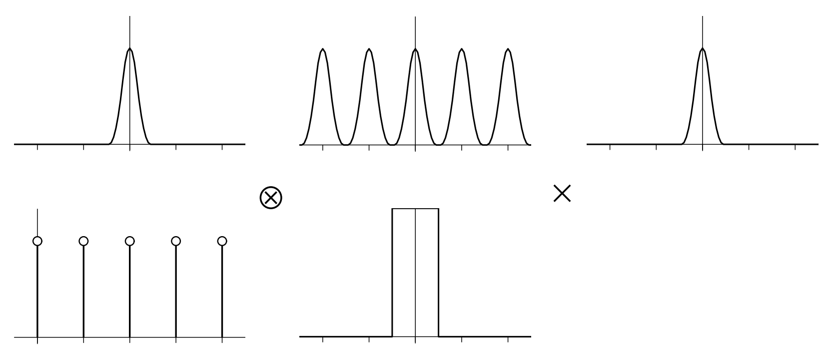
Reconstruction: Frequency Domain



Reconstruction: Spatial Domain



Sampling and Reconstruction



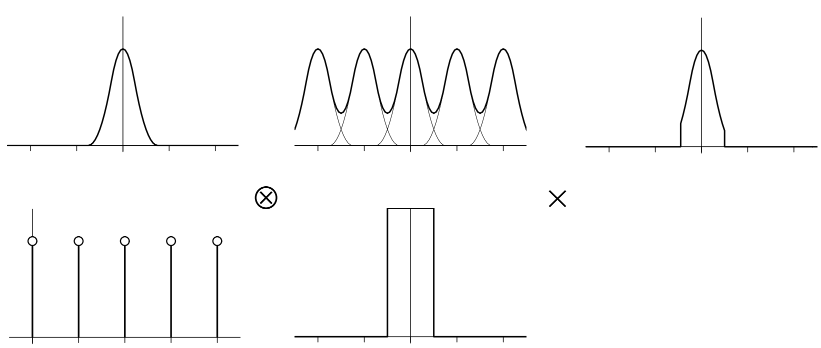
Sampling Theorem

This result if known as the *Sampling Theorem* and is due to Claude Shannon who first discovered it in 1949

> ***A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above 1/2 the sampling frequency***

For a given bandlimited function, the rate at which it must be sampled is called the *Nyquist Frequency*

Undersampling: Aliasing



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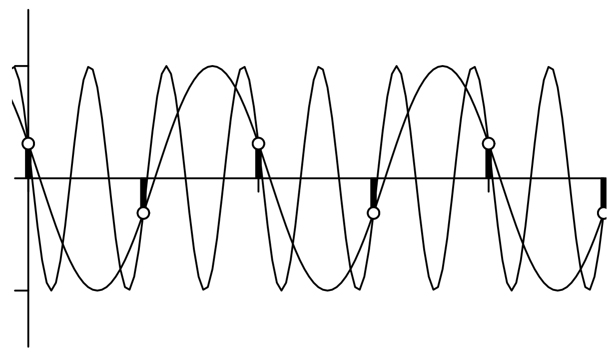
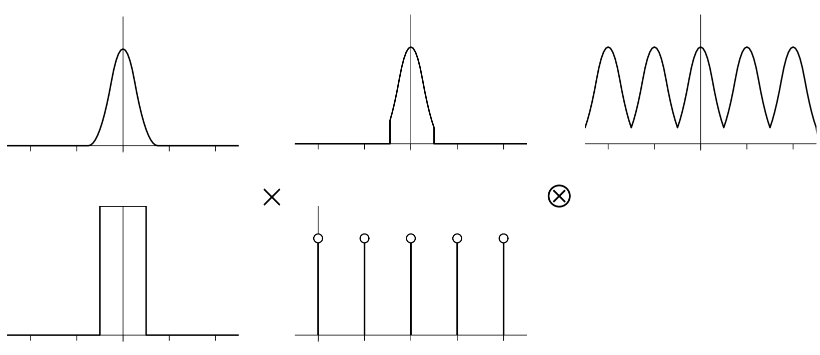
“Aliases”



CS248 Lecture 6

Antialiasing by Prefiltering



CS248 Lecture 6

Antialiasing by Prefiltering

<u>Ideally</u>: perfectly low-pass filter (convolve with a sinc function) to bandlimit the function to the Nyquist sampling rate.

<u>Typically</u>: convolve with a box filter (square pixel area average) or circular filter (circular pixel average) before sampling

Practically:

- Constant colored lines and polygonal fragments doable
- Complex environments not doable