Practical Solutions

Two approaches

- Monte Carlo methods
- Finite element methods

Classic radiosity

- Diffuse, polygonal surfaces
  - View independent solution
  - Polygonal mesh
- Form factors
- Solving linear equations

Examples

Goral

Nishita, Computer room

Cohen, Vermeer

Cohen, Museum

Wallace, Engine room

Lightscape

The Radiosity Equation

Assume diffuse reflection only

Solve for radiosity (2d function)

$$B(x) = B_e(x) + r(x)E(x)$$

$$B(x) = B_e(x) + r(x) \int_{M^2} F(x,x')B(x')\,dA'$$

$$F(x,x') = \frac{G(x,x')}{p}$$

Form factor: The percentage of light leaving ***dA'*** that arrives at ***dA***

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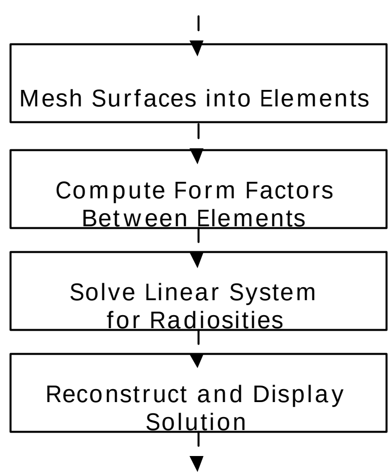
Classic Radiosity Algorithm



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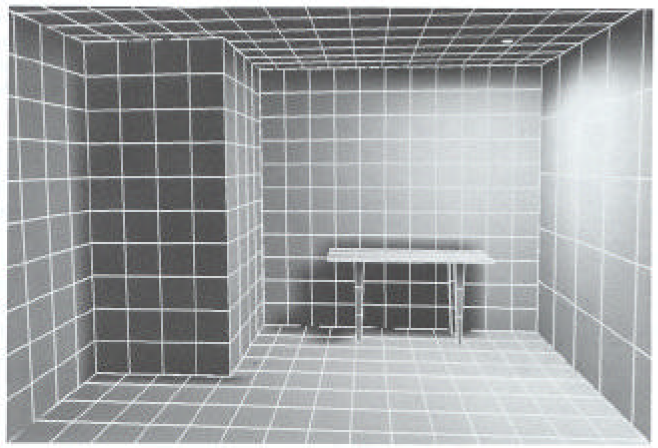
Simple Room Scene



Example from John Wallace

Classic Radiosity

Power Balance

$$B_i A_i = E_i A_i + r_i \sum_j B_j A_j F_{ji}$$

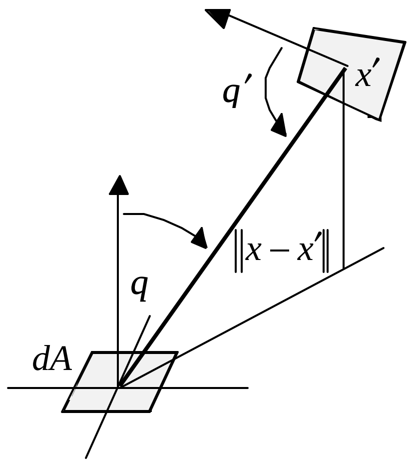
Reciprocity

$$A_i F_{ij} = A_j F_{ji} \Rightarrow B_i = E_i + r_i \sum_j F_{ij} B_j$$

Radiosity System

$$\begin{pmatrix} 1-r_1F_{11} & -r_1F_{12} & \cdots & -r_1F_{1n} \\ -r_2F_{21} & 1-r_2F_{22} & \cdots & -r_2F_{21} \\ \vdots & \vdots & \ddots & \vdots \\ -r_nF_{n1} & -r_nF_{n2} & \cdots & 1-r_nF_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix}$$

Differential Form Factor



$$\cos\theta \, d\omega = \frac{\cos\theta \, \cos\theta'}{\|x-x'\|^2} dA'$$

Form Factors

Differential-differential

$$F_{dA_i, dA_j} = \frac{\cos\theta_o' \cos\theta_i}{\pi \|x-x'\|^2} V(x,x') \, dA_j$$

Differential-finite

$$F_{dA_i, A_j} = \int_{A_j} \frac{\cos\theta_o' \cos\theta_i}{\pi \|x-x'\|^2} V(x,x') \, dA'$$

Finite-finite

$$F_{A_i, A_j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_o' \cos\theta_i}{\pi \|x-x'\|^2} V(x,x') \, dA' \, dA$$

Form Factor

Throughput

$$T_{ij} = T_{ji} = \int_{A_i} \int_{A_j} \frac{\cos q_o' \cos q_i}{p \left\| x - x' \right\|^2} V(x, x')\, dA'\, dA$$

Reciprocity

$$T_{ij} = A_i F_{ij}$$

$$T_{ji} = A_j F_{ji}$$

$$A_i F_{ij} = A_j F_{ji}$$

Form Factor Properties

Summation

$$\sum_j F_{ij} = \sum_i F_{ji} = 1$$

Form factor is the percentage of light...

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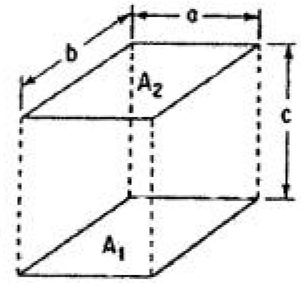
Analytical Form Factors



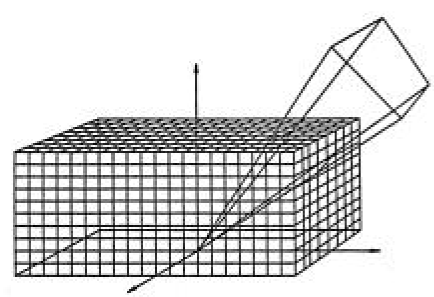
$$X = \frac{a}{c}$$

$$Y = \frac{b}{c}$$

$$F_{A_1,A_2} = \frac{2}{pXY}\left\{ \begin{array}{l} \ln\left[\frac{(1+X^2)(1+Y^2)}{(1+X^2+Y^2)}\right]^{\frac{1}{2}} + X\sqrt{1+Y^2}\tan^{-1}\frac{X}{\sqrt{1+Y^2}} \\ + Y\sqrt{1+X^2}\tan^{-1}\frac{Y}{\sqrt{1+X^2}} - X\tan^{-1}X - Y\tan^{-1}Y \end{array} \right\}$$

Hemicube Algorithm

First radiosity algorithm to deal with occlusion



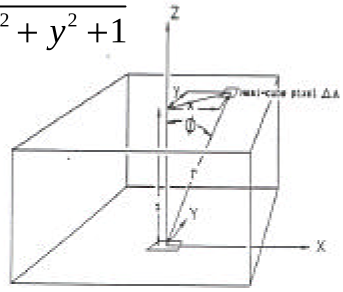
Render source elements from POV of receiving element

$$F_{dA_i,A_j} = \sum_{p \in A_j} \Delta F_p$$

Typical resolution: 32x32

Hemicube Delta Form Factors

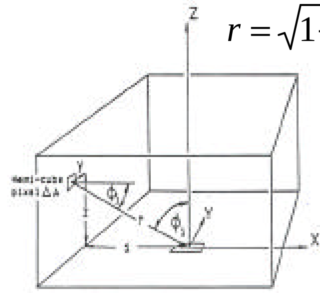
$$r = \sqrt{x^2 + y^2 + 1}$$



$$\cos f = \frac{1}{\sqrt{x^2 + y^2 + 1}}$$

$$\Delta F = \frac{\Delta A}{p\,(x^2 + y^2 + 1)^2}$$

$$r = \sqrt{1 + y^2 + z^2}$$



$$\cos f = \frac{1}{\sqrt{1 + y^2 + z^2}}$$

$$\Delta F = \frac{\Delta A}{p\,(1 + y^2 + z^2)^2}$$

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Hemicube Algorithms

Advantages

+ First practical method -> Patent!
+ Use existing rendering systems; Hardware!
+ Computes all form factors in O(n)

Disadvantages

- Computes differential-finite form factor
- Expensive to compute a single form factor
- Aliasing errors due to sampling
  Randomly rotate/shear hemicube
- Proximity errors
- Visibility errors

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Solve [K][B] = [E]

Direct methods: O(n^3)

- Gaussian elimination

  Goral, Torrance, Greenberg, Battaile, 1984

Iterative methods: O(n^2)

Converge:

Energy conservation -> diagonally dominant

- Gauss-Seidel, Jacobi: <u>Gathering</u>

  Nishita, Nakamae, 1985

  Cohen, Greenberg, 1985

- Southwell: <u>Shooting</u>

  Cohen, Chen, Wallace, Greenberg, 1988

Iterative Solvers

$$(I-F)B=E$$

Iteration

$$B^0=E$$

$$B^1=E+FB^0$$

$$\ldots$$

$$B^n=E+FB^{n-1}$$

Relaxation

Residual

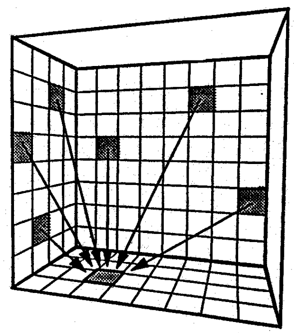
$$r^n=E-(I-F)B^n$$

Iteration

$$r_i^{k+1}=0 \Rightarrow B_i^{k+1}=B_i^k+r_i^k=E_i+\rho_i\sum_{i\neq j}F_{ij}B_j^k$$

If residual is 0, solution has been reached

Gathering



```
for(i=0; i<n; i++)
  B[i] = Be[i];

while( !converged ) {
  for(i=0; i<n; i++) {
     E[i] = 0;
     for(j=0; j<n; j++)
        E[i] += F[i][j] * B[j];
     B[i] = rho[i]*E[i];
  }
}
```

Scan through elements in "model" order

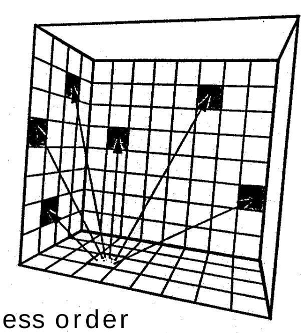
Row of F times B

Successively set residual to 0

May update radiosities at end (Jacobi), or during (GS)

Calculate one row of F and discard

Shooting



```
for(i=0; i<n; i++)
  B[i] = dB[i] = Be[i];

while( !converged ) {
  set i st dB[i] is the largest;
  for(j=0;j<n;j++)
     if(i!=j) {
        dB[j] = rho[j]*F[j][i]*dB[i];
      dB[j] += dBj;
      B[j] += dBj;
    }
  dB[i]=0;
}
```

Brightness order

Column of F times B

In terms of residuals

- Choose element with maximum residual
- Relax such that that elements residual is 0
- Incrementally update other residuals

Results: Gathering vs. Shooting

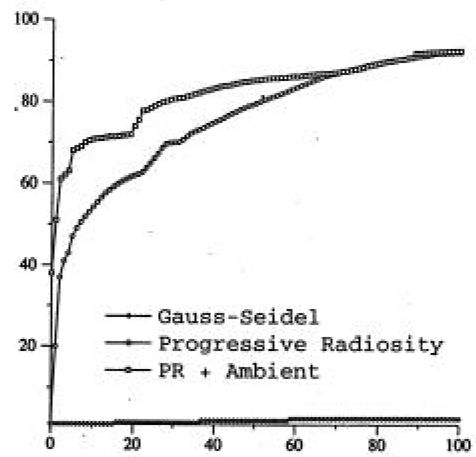


Figure 5.9: *Convergence versus number of steps for three algorithms.*